

Class: XII
SESSION : 2022-2023
SUBJECT: Mathematics
SAMPLE QUESTION PAPER - 1
with SOLUTION

Time Allowed: 3 Hours

Maximum Marks: 80

General Instructions :

1. This Question paper contains - **five sections** A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units of assessment** (4 marks each) with sub parts.

Section A

1. $\int \sqrt{4-x^2} dx = ?$ [1]
a) None of these
b) $\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$
c) $x\sqrt{4-x^2} + \sin^{-1}\frac{x}{2} + C$
d) $\frac{1}{2}x\sqrt{4-x^2} - 2\sin^{-1}\frac{x}{2} + C$
2. $[\hat{i} \quad \hat{j} \quad \hat{k}] = ?$ [1]
a) 3
b) 1
c) 2
d) 0
3. If $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$, then $P\left(\frac{A'}{B'}\right) \cdot P\left(\frac{B'}{A'}\right)$ is equal to [1]
a) $\frac{25}{42}$
b) $\frac{5}{6}$
c) $\frac{5}{7}$
d) 1
4. The angle between two lines having direction ratios 1, 1, 2 and $(\sqrt{3}-1)$, $(-\sqrt{3}-1)$, 4 is [1]
a) $\frac{\pi}{4}$
b) $\frac{\pi}{3}$
c) $\frac{\pi}{6}$
d) $\frac{\pi}{2}$
5. Two numbers are selected at random from integers 1 through 9. If the sum is even, what is the probability that both numbers are odd? [1]
a) $\frac{5}{8}$
b) $\frac{1}{6}$
c) $\frac{4}{9}$
d) $\frac{2}{3}$

6. $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$ is equal to [1]
- a) $\frac{-e^x}{1+x^2} + C$ b) $\frac{e^x}{(1+x^2)} + C$
- c) $\frac{-e^x}{(1+x^2)^2} + C$ d) $\frac{e^x}{(1+x^2)^2} + C$
7. The direction ratios of two lines are 3, 2, -6 and 1, 2, 2 respectively. The acute angle between these lines is [1]
- a) $\cos^{-1} \left(\frac{5}{18} \right)$ b) $\cos^{-1} \left(\frac{8}{21} \right)$
- c) $\cos^{-1} \left(\frac{5}{21} \right)$ d) $\cos^{-1} \left(\frac{3}{20} \right)$
8. The area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ is [1]
- a) $20\pi^2$ sq. units b) 25π sq. units
- c) 20π sq. units d) $16\pi^2$ sq. units
9. If $\vec{a}$ and $\vec{b}$ are unit vectors inclined at an angle θ , then the value of $|\vec{a} - \vec{b}|$ is [1]
- a) $2 \cos \frac{\theta}{2}$ b) $2 \sin \frac{\theta}{2}$
- c) $2 \cos$ d) $2 \sin \theta$
10. The area enclosed by the circle $x^2 + y^2 = 2$ is equal to [1]
- a) $4\pi^2$ sq units b) 4π sq units
- c) 2π sq units d) $2\sqrt{2}\pi$ sq units
11. The degree of the differential equation $\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}} = \frac{d^2y}{dx^2}$ is [1]
- a) 2 b) $\frac{3}{2}$
- c) not defined d) 4
12. $\int \sin^3(2x+1) dx = ?$ [1]
- a) $\frac{1}{2} \cos(2x+1) + \frac{1}{3} \cos^3(2x+1) + C$ b) $-\frac{1}{2} \cos(2x+1) + \frac{1}{6} \cos^3(2x+1) + C$
- c) $\frac{1}{8} \sin^4(2x+1) + C$ d) None of these
13. If $\begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$, and $A + A' = I$, if the value of α is [1]
- a) b)

$$\frac{\pi}{6}$$

$$\frac{\pi}{3}$$

c) $\frac{3\pi}{2}$

d) π

14. Function $f(x) = 2x^3 - 9x^2 + 12x + 29$ is monotonically decreasing when [1]

a) $x > 2$

b) $1 < x < 2$

c) $x < 2$

d) $x > 3$

15. Let A be a non-singular square matrix of order 3×3 . Then $|\text{adj } A|$ is equal to [1]

a) $|A|$

b) $3|A|$

c) $|A|^3$

d) $|A|^2$

16. If A is a 3×3 matrix such that $|A| = 8$, then $|3A|$ equals. [1]

a) 8

b) 72

c) 216

d) 24

17. What is the equation of a curve passing through $(0, 1)$ and whose differential equation is given by $dy = y \tan x \, dx$? [1]

a) $y = \sec x$

b) $y = \sin x$

c) $y = \csc x$

d) $y = \cos x$

18. Domain of $\cos^{-1}x$ is [1]

a) $[-1, 0]$

b) $[0, 1]$

c) None of these

d) $[-1, 1]$

19. **Assertion (A):** If manufacturer can sell x items at a price of $\text{₹}(5 - \frac{x}{100})$ each. The cost price of x items is $\text{₹}(\frac{x}{5} + 500)$. Then, the number of items he should sell to earn maximum profit is 240 items. [1]

Reason (R): The profit for selling x items is given by $\frac{24}{5}x - \frac{x^2}{100} - 300$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular. [1]

Reason (R): A square matrix A is said to be singular, if $|A| = 0$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Write the cofactor of a_{12} in the matrix $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ [2]

OR

Solve the system of equations by matrix method

$$8x + 4y + 3z = 18$$

$$2x + y + z = 5$$

$$x + 2y + z = 5$$

22. Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$. [2]

23. Verify that $y = (a + bx)e^{2x}$ is the general solution of the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$. [2]

24. Given the probability that A can solve a problem is $\frac{2}{3}$, and the probability that B can solve the same problem is $\frac{3}{5}$, find the probability that at least one of A and B will solve the problem. [2]

25. For what value of λ are the vectors $\vec{a}$ and $\vec{b}$ perpendicular to each other? where; $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$ [2]

Section C

26. Find the particular solution of the differential equation $e^x \sqrt{1 - y^2} dx + \frac{y}{x} dy = 0$, given that $y = 1$, when $x = 0$. [3]

OR

Solve the initial value problem: $(x^2 + 1)y' - 2xy = (x^4 + 2x^2 + 1)\cos x$, $y(0) = 0$

27. Prove $\int_0^{\frac{\pi}{4}} 2\tan^3 x dx = 1 - \log 2$ [3]

28. Evaluate: $\int \frac{x^2}{(x^4 - x^2 - 12)} dx$. [3]

OR

Evaluate: $\int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$

29. Show that the points A (1, 2, 7), B (2, 6, 3) and C(3,10, -1) are collinear. [3]

OR

Prove using vectors: The quadrilateral obtained by joining midpoints of adjacent sides of a rectangle is a rhombus.

30. If $x = a(1 - \cos^3 \theta)$, $y = a \sin^3 \theta$, prove that $\frac{d^2y}{dx^2} = \frac{32}{27a}$ at $\theta = \frac{\pi}{6}$ [3]

31. Sketch the region bounded by the curve $y = 2x - x^2$ and the x-axis and find its area. [3]



Section D

32. Minimize $Z = x + 2y$ subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$. Show that the minimum of Z occurs at more than two points. [5]
33. Show that the function $f: R_0 \rightarrow R_0$, defined as $f(x) = \frac{1}{x}$, is one-one onto, where R_0 is the set non-zero real numbers. Is the result true, if the domain R_0 is replaced by N with co-domain being same as R_0 ? [5]

OR

Show that the function $f: R \rightarrow R$ defined by $f(x) = \frac{x}{x^2+1}, \forall x \in R$, is neither one-one nor onto.

34. Find the vector equation of the line passing through $(1, 2, 3)$ and parallel to each of the planes $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$. Also find the point of intersection of the line thus obtained with the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$. [5]

OR

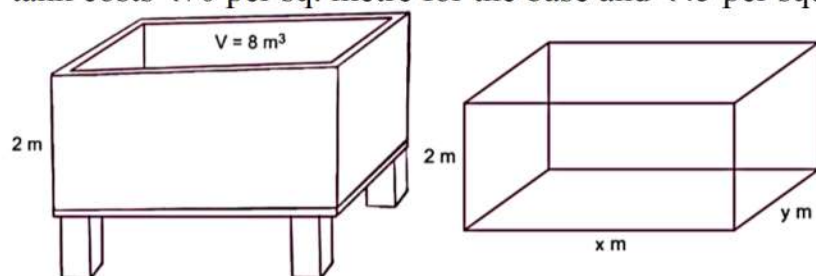
Show that the lines $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$ intersect. Also, find their point intersection.

35. Find $\frac{dy}{dx}$ of the function $x^y + y^x = 1$ [5]

Section E

36. **Read the text carefully and answer the questions:** [4]

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



- (i) Express making cost C in terms of length of rectangle base.
- (ii) If x and y represent the length and breadth of its rectangular base, then find the relation between the variables.
- (iii) Find the value of x so that the cost of construction is minimum.

OR

Verify by second derivative test that cost is minimum at a critical point.

37. **Read the text carefully and answer the questions:** [4]

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A



sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



- (i) Write the matrix summarizing sales data of 2019 and 2020.
- (ii) Find the matrix summarizing sales data of 2020.
- (iii) Find the total number of cars sold in two given years, by each dealer?

OR

If each dealer receives a profit of ₹ 50000 on sale of a Hatchback, ₹100000 on sale of a Sedan and ₹200000 on sale of an SUV, then find the amount of profit received in the year 2020 by each dealer.

38. Read the text carefully and answer the questions:

[4]

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.



- (i) Teacher ask Govind, what is the probability that tickets are drawn by Abhishek, shows a prime number on one ticket and a multiple of 4 on other ticket?
- (ii) Teacher ask Girish, what is the probability that tickets drawn by Ankit, shows an even number on first ticket and an odd number on second ticket?

SOLUTION

Section A

1. (b) $\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C$

Explanation: The given integral is $\int \sqrt{4-x^2} dx$

Using $\int \sqrt{a^2-x^2} dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$

$$\begin{aligned}\int \sqrt{4-x^2} dx &= \int \sqrt{2^2-x^2} dx \\ &= \frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2} + C\end{aligned}$$

2. (b) 1

Explanation: $[\hat{i} \quad \hat{j} \quad \hat{k}] = \hat{i} \cdot (\hat{j} \times \hat{k}) = \hat{i} \cdot \hat{i} \quad \dots (\because \hat{j} \times \hat{k} = \hat{i}) = |\hat{i}|^2 = 1$

3. (a) $\frac{25}{42}$

Explanation: Here, $P(A) = \frac{2}{5}$, $P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$

$$\begin{aligned}P(A'/B') &= \frac{P(A' \cap B')}{P(B')} = \frac{1-P(A \cup B)}{1-P(B)} \\ &= \frac{1-[P(A)+P(B)-P(A \cap B)]}{1-P(B)}\end{aligned}$$

$$= \frac{1-\left(\frac{2}{5} + \frac{3}{10} - \frac{1}{5}\right)}{1-\frac{3}{10}}$$

$$= \frac{1-\left(\frac{4+3-2}{10}\right)}{\frac{7}{10}} = \frac{1-\frac{1}{2}}{\frac{7}{10}} = \frac{5}{7}$$

$$\text{And } P(B'/A') = \frac{P(B' \cap A')}{P(A')} = \frac{1-P(A \cup B)}{1-P(A)}$$

$$= \frac{1 - \frac{1}{2}}{1 - \frac{1}{5}} = \frac{1/2}{3/5} = \frac{5}{6} \left[\because P(A \cup B) = \frac{1}{2} \right]$$

$$\therefore P(A' / B') \cdot P(B' / A') = \frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}$$

4. (d) $\frac{\pi}{2}$

Explanation: Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = (\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k}$

$$|\vec{a}| = \sqrt{6}, |\vec{b}| = \sqrt{(4 - 2\sqrt{3}) + (4 + 2\sqrt{3}) + 16} = 2\sqrt{6}$$

$$\cos \alpha = \frac{(\hat{i} + \hat{j} + 2\hat{k}) \cdot ((\sqrt{3} - 1)\hat{i} + (-\sqrt{3} - 1)\hat{j} + 4\hat{k})}{\sqrt{6} \times 2\sqrt{6}}$$

$$\cos \alpha = \frac{\sqrt{3} - 1 - \sqrt{3} - 1 + 8}{12}$$

$$\cos \alpha = \frac{1}{2}$$

$$\alpha = 60^\circ$$

5. (a) $\frac{5}{8}$

Explanation: The sum will be even when; both numbers are either even or odd, i.e. for both numbers to be even, the total cases ${}^5C_1 \times {}^4C_1$ (Both the numbers are odd) + ${}^4C_1 \times {}^3C_1$ (Both the numbers are even) = 32

The favourable number of cases will be,

Both odd, i.e. selecting numbers from 1, 3, 5, 7, or 9, i.e.

$${}^5C_1 \times {}^4C_1 = 20$$

Thus, the probability that both numbers are odd will be

$$= \frac{\text{Favorable outcomes}}{\text{Total outcomes}}$$

$$\Rightarrow \frac{20}{32} = \frac{5}{8}$$

6. (b) $\frac{e^x}{(1+x^2)} + C$

Explanation: Given $\int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx$

$$\Rightarrow \int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx = \int e^x \left(\frac{1+x^2-2x}{(1+x^2)^2} \right) dx$$

$$\Rightarrow \int e^x \left(\frac{1+x^2-2x}{(1+x^2)^2} \right) dx = \int e^x \left\{ \left(\frac{1+x^2}{(1+x^2)^2} \right) + \left(\frac{-2x}{(1+x^2)^2} \right) \right\} dx$$

$$= \int e^x \left\{ \left(\frac{1}{1+x^2} \right) + \left(\frac{-2x}{(1+x^2)^2} \right) \right\} dx$$

Now using the property: $\int e^x (f(x) + f'(x)) dx = e^x f(x)$

$$\text{Now in } \int e^x \left\{ \left(\frac{1}{1+x^2} \right) + \left(\frac{-2x}{(1+x^2)^2} \right) \right\} dx$$

$$\Rightarrow f(x) = \frac{1}{1+x^2}$$

$$\Rightarrow f'(x) = \frac{-2x}{(1+x^2)^2}$$

$$\Rightarrow \int e^x \left\{ \left(\frac{1}{1+x^2} \right) + \left(\frac{-2x}{(1+x^2)^2} \right) \right\} dx = \frac{e^x}{1+x^2} + C$$

$$\Rightarrow \int e^x \left(\frac{1-x}{1+x^2} \right)^2 dx = \frac{e^x}{1+x^2} + C.$$

Which is the required solution.

7. (c) $\cos^{-1} \left(\frac{5}{21} \right)$

Explanation: Direction ratios are given implies that we can write the parallel vector towards that line, let's consider first parallel vector to be $\vec{a} = 3\hat{i} + 2\hat{j} - 6\hat{k}$ and second parallel vector be $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

For the angle, we can use the formula $\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$

For that, we need to find the magnitude of these vectors

$$|\vec{a}| = \sqrt{3^2 + 2^2 + (-6)^2}$$

$$= 7$$

$$|\vec{b}| = \sqrt{1 + 2^2 + 2^2}$$

$$= 3$$

$$\Rightarrow \cos \alpha = \frac{(3\hat{i} + 2\hat{j} - 6\hat{k}) \cdot (\hat{i} + 2\hat{j} + 2\hat{k})}{7 \times 3}$$

$$\Rightarrow \cos \alpha = \frac{3 + 4 - 12}{21}$$

$$\Rightarrow \cos \alpha = \frac{-5}{21}$$

$$\therefore \alpha = \cos^{-1} \left(-\frac{5}{21} \right)$$

The negative sign does not affect anything in cosine as cosine is positive in the fourth quadrant.

$$\alpha = \cos^{-1} \left(\frac{5}{21} \right)$$

8. (c) 20π sq. units

Explanation: The area of the standard ellipse is given by ; πab . Here, $a = 5$ and $b = 4$
Therefore, the area of curve is $\pi(5)(4) = 20\pi$.

9. (b) $2\sin^2 \frac{\theta}{2}$

Explanation: Given $\vec{a}$ and $\vec{b}$ are unit vectors with inclination is θ

$$\text{now, } |\vec{a} - \vec{b}|^2 = (\vec{a} - \vec{b}) \cdot (\vec{a} - \vec{b}) = |\vec{a}|^2 - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{a} + |\vec{b}|^2$$

$$= 1 - 2(\vec{a} \cdot \vec{b}) + 1$$

$$= 2 - 2|\vec{a}| |\vec{b}| \cos \theta$$

$$= 2 - 2\cos \theta \quad (\text{where vectors are unit vectors})$$

$$= 2(1 - \cos \theta)$$

$$= 4\sin^2 \frac{\theta}{2}$$

$$\text{thus } |\vec{a} - \vec{b}|^2 = 4\sin^2 \frac{\theta}{2}$$

$$\therefore |\vec{a} - \vec{b}| = 2\sin \frac{\theta}{2}$$

10. (c) 2π sq units

Explanation: Since Area = $4 \int_0^{\sqrt{2}} \sqrt{2-x^2} dx$

$$= 4 \left(\frac{x}{2} \sqrt{2-x^2} + \sin^{-1} \frac{x}{\sqrt{2}} \right) \sqrt{2} \Big|_0^{\sqrt{2}} = 2\pi \text{ sq. units}$$

11. (a) 2

Explanation: In general terms for a polynomial the degree is the highest power.

The differential equation is $\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = \frac{d^2y}{dx^2}$

Square both the sides

$$\Rightarrow \left(1 + \left(\frac{dy}{dx}\right)^2\right)^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

Now for degree to exist the given differential equation must be a polynomial in some differentials.

Here differentials mean $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$ or $\frac{d^ny}{dx^n}$

The given differential equation is polynomial in differentials $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Degree of differential equation is defined as the highest integer power of highest order derivative in the equation.

Here the highest derivative is $\frac{d^2y}{dx^2}$ and there is only one term of highest order derivative in

the equation which is $\left(\frac{d^2y}{dx^2}\right)^2$ whose power is 2 hence degree is 2.

12. (b) $-\frac{1}{2}\cos(2x+1) + \frac{1}{6}\cos^3(2x+1) + C$

Explanation: Formula :- $\int x^n dx = \frac{x^{n+1}}{n+1} + c$; $\int \frac{1}{1+x^2} dx = \tan^{-1}x + c$

Therefore ,

$$\Rightarrow \int \sin^2(2x+1) \sin(2x+1) dx = \int (1 - \cos^2(2x+1)) \sin(2x+1) dx$$

$$= \int \sin(2x+1) dx - \int \cos^2(2x+1) \sin(2x+1) dx$$

Put $\cos(2x+1) = t$

$$\Rightarrow -2\sin(2x+1) dx = dt$$

$$I = -\int \frac{dt}{2} - \left(-\frac{1}{2}\right) \int t^2 dt$$

$$= -\frac{1}{2} \int dt + \frac{1}{2} \int t^2 dt$$

$$= -\frac{1}{2}t + \frac{1}{2} \frac{t^3}{3} + c$$

$$= -\frac{1}{2}t + \frac{t^3}{6} + c$$

$$= -\frac{1}{2}\cos(2x+1) + \frac{[\cos(2x+1)]^3}{6} + c$$

13. (b) $\frac{\pi}{3}$

Explanation: Given $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Therefore, $A' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Also given that $A + A' = I \dots(1)$

(Putting the values in equation (1))

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We know the two matrices are equal only when all their corresponding elements or entries are equal i.e. if $A = B$, then a_{ij} and b_{ij} for all i and j .

This implies,

$$2\cos\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{2}$$

$$\Rightarrow \cos\alpha = \cos\frac{\pi}{3} \dots \left(\because \cos\frac{\pi}{3} = \frac{1}{2} \right)$$

$$\Rightarrow \alpha = \frac{\pi}{3}$$

14. (b) $1 < x < 2$

Explanation: $1 < x < 2$

15. (d) $|A|^2$

Explanation: For a square matrix of order $n \times n$,

We know that $A \cdot \text{adj}A = |A|I$

Here, $n=3$

$$\therefore |A \cdot \text{adj}A| = |A|^n$$

$$|\text{adj}A| = |A|^{n-1}$$

$$\text{So, } |\text{Adj}A| = |A|^{3-1} = |A|^2$$

16. (c) 216

Explanation: Given A is a square matrix of order 3 and also $|A| = 8$

$$|3A| = (3)^3 \times |A| = 27 \times 8 = 216$$

17. (a) $y = \sec x$

Explanation: The given differential equation of the curve is,

$$dy = y \tan x \, dx \Rightarrow \int \frac{dy}{y} = \int \tan x \cdot dx \text{ [on integrating]}$$

$$\Rightarrow \log y = \log \sec x + \log C \Rightarrow \log y = \log C \sec x$$

$$\Rightarrow y = C \sec x \dots (i)$$

Since, the curve passes through the origin (0, 1), then

$$1 = C \sec 0 \Rightarrow C = 1$$

$\therefore$ Required equation of curve is, $y = \sec x$

18. (d) $[-1, 1]$

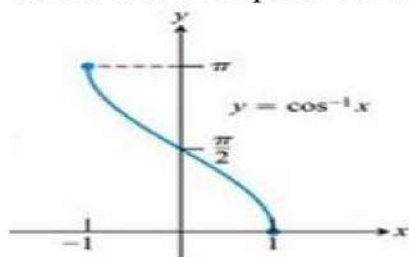
Explanation: To Find: The range of $\cos^{-1}(x)$

Here, the inverse function is given by $y = f^{-1}(x)$

The graph of the function $y = \cos^{-1}(x)$ can be obtained from the graph of

$Y = \cos x$ by interchanging x and y axes. i.e, if a, b is a point on $Y = \cos x$ then b, a is the point on the function $y = \cos^{-1}(x)$

Below is the Graph of the range of $\cos^{-1}(x)$



From the graph, it is clear that the domain of $\cos^{-1}(x)$ is $[-1, 1]$

19. (c) A is true but R is false.

Explanation: Let $S(x)$ be the selling price of x items and let $C(x)$ be the cost price of x items.

Then, we have

$$S(x) = \left(5 - \frac{x}{100}\right)x = 5x - \frac{x^2}{100}$$

$$\text{and } C(x) = \frac{x}{5} + 500$$

Thus, the profit function $P(x)$ is given by

$$P(x) = S(x) - C(x) = 5x - \frac{x^2}{100} - \frac{x}{5} - 500$$

$$\text{i.e. } P(x) = \frac{24}{5}x - \frac{x^2}{100} - 500$$

On differentiating both sides w.r.t. x , we get

$$P'(x) = \frac{24}{5} - \frac{x}{50}$$

Now, $P'(x) = 0$ gives $x = 240$.

$$\text{Also, } P'(x) = \frac{-1}{50}.$$

$$\text{So, } P'(240) = \frac{-1}{50} < 0$$

Thus, $x = 240$ is a point of maxima.

Hence, the manufacturer can earn maximum profit, if he sells 240 items.

20. (a) Both A and R are true and R is the correct explanation of A.

Explanation: The determinant of the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix} = 8 - 8 = 0$

Hence, A is a singular matrix.

Section B

21. Given the matrix is, $\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

We need to find the cofactor of a_{12} in the matrix

$$\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$$

Firstly we know what the element at position a_{12} in the matrix is.

$$a_{12} = -3$$

And as discussed above, the sign at a_{12} is (-).

For cofactor of -3, eliminate first row and second column in the matrix.

$$\text{Cofactor of } -3 = \begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix}$$

$$\Rightarrow \text{Cofactor of } -3 = (6 \times -7) - (4 \times 1)$$

$$\Rightarrow \text{Cofactor of } -3 = -42 - 4$$

$$\Rightarrow \text{Cofactor of } -3 = -46$$

Since, the sign of cofactor of -3 is (-), then

$$\text{Cofactor of } -3 = -(-46)$$

$$\Rightarrow \text{Cofactor of } -3 = 46$$

Thus, the cofactor of -3 is 46.

OR

Given system of equations

$$8x + 4y + 3z = 18$$

$$2x + y + z = 5$$

$$x + 2y + z = 5$$

The given system can be re-written in matrix form as:

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} X \\ y \\ z \end{bmatrix} = \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$AX = B$$

$$\begin{aligned} \text{Now, } |A| &= 8 \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} - 4 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} \\ &= 8(-1) - 4(1) + 3(3) \\ &= -8 - 4 + 9 \\ &= -3 \end{aligned}$$

So, the above system has a unique solution, given by

$$X = A^{-1}B$$

Cofactors of A are:

$$C_{11} = (-1)^{1+1} 1 - 2 = -1$$

$$C_{21} = (-1)^{2+1} 4 - 6 = 2$$

$$C_{31} = (-1)^{3+1} 4 - 3 = 1$$

$$C_{12} = (-1)^{1+2} 2 - 1 = -1$$

$$C_{22} = (-1)^{2+2} 8 - 3 = 5$$

$$C_{32} = (-1)^{3+2} 8 - 6 = -2$$

$$C_{13} = (-1)^{1+3} 4 - 1 = 3$$

$$C_{23} = (-1)^{2+3} 16 - 4 = -12$$

$$C_{33} = (-1)^{3+3} 8 - 8 = 0$$

$$\text{adj}A = \begin{bmatrix} -1 & -1 & 3 \\ 2 & 5 & -12 \\ 1 & -2 & 0 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj}A$$

$$\text{Now, } X = A^{-1}B = \frac{1}{-3} \begin{bmatrix} -1 & 2 & 1 \\ -1 & 5 & -2 \\ 3 & -12 & 0 \end{bmatrix} \begin{bmatrix} 18 \\ 5 \\ 5 \end{bmatrix}$$

$$x = \frac{1}{3} \begin{bmatrix} -18 + 10 + 5 \\ -18 + 25 - 10 \\ 54 - 60 + 0 \end{bmatrix}$$

$$x = \frac{1}{-3} \begin{bmatrix} -3 \\ -3 \\ -6 \end{bmatrix}$$

$$x = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

Hence, $X = 1, Y = 1$ and $Z = 2$

22. We have, $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(\frac{-\pi}{2}\right)\right]$.

$$= \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}(-1).$$

$$= \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{6}\right)\right] + \cot^{-1}\left[\cot\left(\frac{\pi}{3}\right)\right] + \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right) + \cot^{-1}\left(\cot\frac{\pi}{3}\right) + \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$$

$$\left[\begin{array}{l} \because \tan^{-1}(\tan x) = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \\ \cot^{-1}(\cot x) = x, x \in (0, \pi) \\ \text{and } \tan^{-1}(-x) = -\tan^{-1}x \end{array} \right]$$

$$= -\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} = \frac{-2\pi + 4\pi - 3\pi}{12}$$

$$= \frac{-5\pi + 4\pi}{12} = \frac{-\pi}{12}$$

23. $y = (a + bx)e^{2x}$

Differentiating above equation with respect to x , we get

$$\frac{dy}{dx} = be^{2x} + 2(a + bx)e^{2x}$$

On differentiating again w.r.t. x , we get

$$\frac{d^2y}{dx^2} = 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x}$$

$$\text{Now, } \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y$$

$$= 2be^{2x} + 2be^{2x} + 4(a + bx)e^{2x} - 4be^{2x} - 8(a + bx)e^{2x} + 4(a + bx)e^{2x} = 0$$

$$y = (a + bx)e^{2x} \text{ is the solution of } \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

24. Given : Here probability of A and B that can solve the same problem is given , i.e., $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5} \Rightarrow P(\bar{A}) = \frac{1}{3}$ and $P(\bar{B}) = \frac{2}{5}$

Also, A and B are independent . not A and not B are independent.

To Find: atleast one of A and B will solve the problem

Now , $P(\text{atleast one of them will solve the problem}) = 1 - P(\text{both are unable to solve})$

$$= 1 - P(\bar{A} \cap \bar{B})$$

$$= 1 - P(\bar{A}) \times P(\bar{B})$$

$$= 1 - \left(\frac{1}{3} \times \frac{2}{5} \right)$$

$$= \frac{13}{15}$$

25. Since, $\vec{a}$ and $\vec{b}$ are perpendicular

$$\therefore \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow (\lambda\hat{i} + 2\hat{j} + \hat{k}) \cdot (4\hat{i} - 9\hat{j} + 2\hat{k}) = 0$$

$$\Rightarrow (\lambda)(4) + (2)(-9) + (1)(2) = 0$$

$$\Rightarrow 4\lambda - 18 + 2 = 0$$

$$\Rightarrow 4\lambda - 16 = 0$$

$$\Rightarrow 4\lambda = 16$$

$$\Rightarrow \lambda = \frac{16}{4}$$

$$\Rightarrow \lambda = 4$$

Section C

26. We have,

$$e^x \sqrt{1-y^2} dx + \frac{y}{x} dy = 0$$

$$\Rightarrow e^x \sqrt{1-y^2} dx = \frac{-y}{x} dy$$

Therefore, on separating the variables, we get,

$$\frac{-y}{\sqrt{1-y^2}} dy = xe^x dx$$

Therefore, on integrating both sides, we get,

$$\int \frac{-y}{\sqrt{1-y^2}} dy = \int xe^x dx$$

On putting $1 - y^2 = t \Rightarrow -y dy = \frac{dt}{2}$ in LHS, we get

$$\int \frac{1}{2\sqrt{t}} dt = \int x e^x dx$$

III

$$\Rightarrow \frac{1}{2}[2\sqrt{t}] = x \int e^x dx - \int \left[\frac{d}{dx}(x) \int e^x dx \right] dx \text{ [using integration by parts]}$$

$$\Rightarrow \sqrt{1 - y^2} = x e^x - \int e^x dx \quad \left[\text{put } t = 1 - y^2 \right]$$

$$\Rightarrow \sqrt{1 - y^2} = x e^x - e^x + C \dots (i)$$

Also, given that $y = 1$, when $x = 0$

On putting $y = 1$ and $x = 0$ in Eq. (i), we get

$$\sqrt{1 - 1} = 0 - e^0 + C$$

$$\Rightarrow C = 1 \quad \left[\because e^0 = 1 \right]$$

On substituting the value of C in Eq. (i), we get

$$\sqrt{1 - y^2} = x e^x - e^x + 1$$

which is the required particular solution of given differential equation.

OR

The given differential equation is:

$$(x^2 + 1) y' - 2xy = (x^4 + 2x^2 + 1) \cos x$$

$$\Rightarrow \frac{dy}{dx} - \frac{2x}{x^2 + 1} y = (x^2 + 1) \cos x \dots (i)$$

This is a linear differential equation with $P = \frac{-2x}{x^2 + 1}$ and $Q = (x^2 + 1) \cos x$

$$\therefore \text{I.F.} = e^{\int \frac{-2x}{x^2 + 1} dx} = e^{-\log(x^2 + 1)} = (x^2 + 1)^{-1}$$

Multiplying (i) by $\frac{1}{x^2 + 1}$, we get

$$\frac{1}{x^2 + 1} \frac{dy}{dx} - \frac{2x}{(x^2 + 1)^2} y = \cos x$$

Integrating both sides with respect to x , we get

$$y \times \frac{1}{x^2 + 1} = \int \cos x dx + C$$

$$\Rightarrow \frac{y}{x^2 + 1} = \sin x + C \dots (ii)$$

It is given that $y(0) = 0$ i.e. $y = 0$ when $x = 0$

Put $x = 0$, $y = 0$ in (ii), we get: $C = 0$

Put $C = 0$ in (ii), we get

$$\frac{y}{x^2 + 1} = \sin x \Rightarrow y = (x^2 + 1) \sin x.$$

27. Given integral is: $\int_0^{\frac{\pi}{4}} 2 \tan^3 x dx$

$$\text{To Prove: } \int_0^{\frac{\pi}{4}} 2 \tan^3 x dx = 1 - \log 2$$

$$\text{Let } I = \int_0^{\frac{\pi}{4}} 2 \tan^3 x dx \dots (i)$$

$$= \int_0^{\frac{\pi}{4}} 2 \cdot \tan x \cdot \tan^2 x dx$$

$$= 2 \cdot \int_0^{\frac{\pi}{4}} \tan x \cdot (\sec^2 x - 1) dx$$

$$\Rightarrow I = 2 \left\{ -\int_0^{\frac{\pi}{4}} \tan x dx + \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx \right\}$$

$$\Rightarrow I = -[2 \log \cos x]_0^{\pi/4} + 2 \cdot I_1 \dots (ii)$$

Solving I_1 :

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx$$

$$\Rightarrow I_1 = \int_0^{\frac{\pi}{4}} \tan x \cdot \sec^2 x dx$$

$$\text{Let, } \tan x = t \Rightarrow \sec^2 x dx = dt$$

$$\text{When } x = 0 \text{ then } t = 0 \text{ and when } x = \frac{\pi}{4} \text{ then } t = 1$$

$$\Rightarrow I_1 = \int_0^1 t dt$$

$$= \left[\frac{t^2}{2} \right]_0^1$$

$$\Rightarrow I_1 = \frac{1}{2}$$

Using this in equation (ii)

$$\Rightarrow I = [2 \log \cos x]_0^{\pi/4} + 2 \cdot \frac{1}{2}$$

$$\Rightarrow I = 2 \left\{ \log \cos \frac{\pi}{4} - \log \cos 0 \right\} + 1$$

$$\Rightarrow I = 2 \left\{ \log \frac{1}{\sqrt{2}} - \log 1 \right\} + 1$$

$$\Rightarrow I = \left\{ \log \left(\frac{1}{\sqrt{2}} \right)^2 - \log(1)^2 \right\} + 1$$

$$\Rightarrow I = 1 - \log 2 + \log 1$$

$$\Rightarrow I = 1 - \log 2$$

Hence Proved.

28. Let, $I = \int \frac{x^2}{(x^4 - x^2 - 12)} dx$

Using partial fractions,

$$\frac{x^2}{(x^4 - x^2 - 12)} = \frac{t}{t^2 - t - 12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3} \dots (1)$$

Where $t = x^2$

$$A(t+3) + B(t-4) = t$$

Now put $t+3=0$

$$t = -3$$

$$A(0) + B(-7) = -3$$

$$B = \frac{3}{7}$$

Now put $t-4=0$

$$t = 4$$

$$A(4+3) + B(0) = 4$$

$$A = \frac{4}{7}$$

From equation(1)

$$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$$

$$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7} \times \frac{1}{x^2-2^2} + \frac{3}{7} \times \frac{1}{x^2+(\sqrt{3})^2}$$

$$\int \frac{x^2}{(x^2-4)(x^2+3)} dx = \frac{4}{7} \int \frac{1}{x^2-2^2} dx + \frac{3}{7} \int \frac{1}{x^2+(\sqrt{3})^2} dx$$

$$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} \log \left| \frac{x-2}{x+2} \right| + \frac{3}{7} \times \frac{1}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} + c$$

$$= \frac{1}{7} \log \left| \frac{x-2}{x+2} \right| + \frac{\sqrt{3}}{7} \tan^{-1} \frac{x}{\sqrt{3}} + c$$

OR

Let the given integral be,

$$I = \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

$$\text{Let } x = a \tan^2 \theta$$

$$dx = 2a \tan \theta \sec^2 \theta d\theta$$

$$I = \left(\sin^{-1} \sqrt{\frac{a \tan^2 \theta}{a + a \tan^2 \theta}} \right) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int \left(\sin^{-1} \sqrt{\frac{\tan^2 \theta}{\sec^2 \theta}} \right) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= \int \sin^{-1}(\sin \theta) (2a \tan \theta \sec^2 \theta) d\theta$$

$$= 2a \int \theta (\tan \theta \sec^2 \theta) d\theta$$

$$= 2a \left[\theta \int \tan \theta \sec^2 \theta d\theta - \int \left(\int \tan \theta \sec^2 \theta d\theta \right) d\theta \right]$$

$$= 2a \left[\theta \frac{\tan^2 \theta}{2} - \int \frac{\tan^2 \theta}{2} d\theta \right]$$

$$= a\theta \tan^2 \theta - \frac{2a}{2} \int (\sec^2 \theta - 1) d\theta$$

$$= a\theta \tan^2 \theta - a \tan \theta + a\theta + c$$

$$= a \left(\tan^{-1} \sqrt{\frac{x}{a}} \right) \frac{x}{a} - a \sqrt{\frac{x}{a}} + a \tan^{-1} \sqrt{\frac{x}{a}} + c$$

$$I = x \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + a \tan^{-1} \sqrt{\frac{x}{a}} + c$$

29. The given points are A (1, 2, 7), B (2, 6, 3) and C(3,10, - 1) respectively.

$$\therefore \text{Position vector of point } A = \vec{OA} = \hat{i} + 2\hat{j} + 7\hat{k}$$

$$\text{Position vector of point } B = \vec{OB} = 2\hat{i} + 6\hat{j} + 3\hat{k}$$

$$\text{Position vector of point } C = \vec{OC} = 3\hat{i} + 10\hat{j} - \hat{k}$$

$$\text{Now } \vec{AB} = \text{Position vector of point B} - \text{Position vector of point A}$$

$$= 2\hat{i} + 6\hat{j} + 3\hat{k} - (\hat{i} + 2\hat{j} + 7\hat{k})$$

$$= 2\hat{i} + 6\hat{j} + 3\hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = \hat{i} + 4\hat{j} - 4\hat{k} \dots (i)$$

$$\text{And } \vec{AC} = \text{Position vector of point C} - \text{Position vector of point A}$$

$$= 3\hat{i} + 10\hat{j} - \hat{k} - (\hat{i} + 2\hat{j} + 7\hat{k})$$

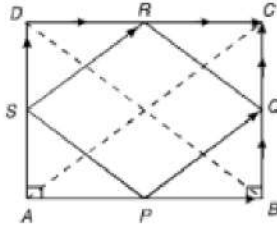
$$= 3\hat{i} + 10\hat{j} - \hat{k} - \hat{i} - 2\hat{j} - 7\hat{k} = 2\hat{i} + 8\hat{j} - 8\hat{k} = 2(\hat{i} + 4\hat{j} - 4\hat{k}) \dots (ii)$$

$$\Rightarrow \vec{AC} = 2\vec{AB} \text{ [Using eq. (i)]}$$

$$\Rightarrow \text{Vectors } \vec{AB} \text{ and } \vec{AC} \text{ are parallel. [} \because \vec{a} = m\vec{b} \text{]}$$

But $\vec{AB}$ and $\vec{AC}$ have a common point A and hence they can't be parallel. Thus, the points A, B, C are collinear.

OR



ABCD be rectangle.

Let P, Q, R and S be the midpoints of the sides AB, BC, CD and DA respectively,
Now

$$\vec{PQ} = \vec{PB} + \vec{BQ} = \frac{1}{2}(\vec{AB} + \vec{BC}) = \frac{1}{2}\vec{AC} \dots (i)$$

$$\vec{SR} = \vec{SD} + \vec{DR} = \frac{1}{2}(\vec{AD} + \vec{DC}) = \frac{1}{2}\vec{AC} \dots (ii)$$

From (i) and (ii) we have

$\vec{PQ} = \vec{SR}$ i. e. sides PQ and SR are equal and parallel
PQRS is a parallelogram.

$$(\vec{PQ})^2 = \vec{PQ} \cdot \vec{PQ} = (\vec{PB} + \vec{BQ}) \cdot (\vec{PB} + \vec{BQ}) = |\vec{PB}|^2 + |\vec{BQ}|^2 \dots (iii)$$

$$(\vec{PS})^2 = \vec{PS} \cdot \vec{PS} = (\vec{PA} + \vec{AS}) \cdot (\vec{PA} + \vec{AS}) = |\vec{PA}|^2 + |\vec{AS}|^2 = |\vec{PB}|^2 + |\vec{BQ}|^2 \dots (iv)$$

From (iii) and (iv) we get,

$(\vec{PQ})^2 = (\vec{PS})^2$ i. e. $PQ = PS$
= The adjacent sides of PQRS are equal.
PQRS is a rhombus.

30. Given,

$$x = a(1 - \cos^3 \theta) \dots (i)$$

$$y = a \sin^3 \theta, \dots (ii)$$

$$\text{To prove: } \frac{d^2y}{dx^2} = \frac{32}{27a} \text{ at } \theta = \frac{\pi}{6}$$

To find the above we will differentiate the function y wrt x twice.

$$\text{As } \frac{d^2y}{dx^2} = \frac{d}{dx} \left(\frac{dy}{dx} \right)$$

So, let's first find dy/dx using parametric form and differentiate it again.

$$\frac{dx}{d\theta} = \frac{d}{d\theta} a(1 - \cos^3 \theta) = 3a \cos^2 \theta \sin \theta \dots \text{(iii)} \text{ [using chain rule]}$$

Similarly,

$$\frac{dy}{d\theta} = \frac{d}{d\theta} a \sin^3 \theta = 3a \sin^2 \theta \cos \theta \dots \text{(iv)}$$

$$\left[\because \frac{d}{dx} \cos x = -\sin x \& \frac{d}{dx} \sin x = \cos x \right]$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a \sin^2 \theta \cos \theta}{3a \cos^2 \theta \sin \theta} = \tan \theta$$

Differentiating again w.r.t. x:

$$\frac{d}{dx} \left(\frac{dy}{dx} \right) = \frac{d}{dx} (\tan \theta)$$

$$\frac{d^2 y}{dx^2} = \sec^2 \theta \frac{d\theta}{dx} \dots \text{(v)}$$

[using chain rule and $\frac{d}{dx} \tan x = \sec^2 x$]

From equation (iii)

$$\frac{dx}{d\theta} = 3a \cos^2 \theta \sin \theta$$

$$\therefore \frac{d\theta}{dx} = \frac{1}{3a \cos^2 \theta \sin \theta}$$

Putting the value in equation ... (v)

$$\frac{d^2 y}{dx^2} = \sec^2 \theta \frac{1}{3a \cos^2 \theta \sin \theta}$$

$$\frac{d^2 y}{dx^2} = \frac{1}{3a \cos^4 \theta \sin \theta}$$

Put $\theta = \pi/6$

$$\left(\frac{d^2 y}{dx^2} \right) \text{ at } \left(x = \frac{\pi}{6} \right) = \frac{1}{3a \cos^4 \frac{\pi}{6} \sin \frac{\pi}{6}} = \frac{1}{3a \left(\frac{\sqrt{3}}{2} \right)^4 \frac{1}{2}}$$

$$\therefore \left(\frac{d^2 y}{dx^2} \right) \text{ at } \left(x = \frac{\pi}{6} \right) = \frac{32}{27a}$$

Hence proved

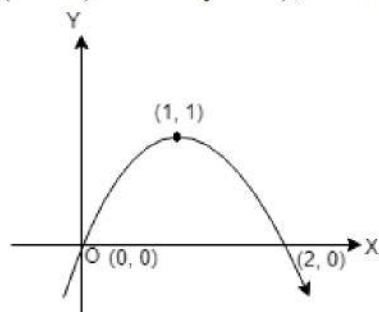
31. The equation of curve is

$$y = 2x - x^2 \dots\dots\dots(1)$$

$$\Rightarrow x^2 - 2x = -y$$

$$\Rightarrow x^2 - 2x + 1 = -y + 1$$

$(x - 1)^2 = -(y - 1)$, which is a downward parabola with vertex (1,1).



putting $y = 0$ in (1), we get, $0 = 2x - x^2$,

$$\therefore x(x - 2) = 0$$

$$x = 0, 2$$

$\therefore$ parabola meets x-axis at (0,0),(2,0)

$\therefore$ required area= Area bounded by the curve $y = 2x - x^2$ and the x-axis

$$= \int_0^2 y \, dx$$

$$= \int_0^2 (2x - x^2) \, dx$$

$$= \left[x^2 - \frac{x^3}{3} \right]_0^2$$

$$= \left[4 - \frac{8}{3} - (0 - 0) \right]$$

$$= \frac{4}{3} \text{ sq units}$$

Section D

32. Consider $2x + y \geq 3$

$$\text{Let } 2x + y = 3 \Rightarrow y = 3 - 2x$$

x	0	1	-1
y	3	1	5

(0, 0) is not contained in the required half plane as (0, 0) does not satisfy the inequation $2x + y \geq 3$.

Again $x + 2y \geq 6$

$$\text{Let } x + 2y = 6$$

$$\Rightarrow \frac{x}{6} + \frac{y}{3} = 1$$

Here also (0, 0) does not contain the required half plane. The double shaded region XABY is the solution set. Its corners are A (6, 0) and B (0, 3).

$$\text{At A (6, 0) } Z = 6 + 0 = 6$$

$$\text{At B (0, 3) } Z = 0 + 2 \times 3 = 6$$

Therefore, at both points the value of $Z = 6$ which is minimum. In fact at every point on the line AB makes $Z = 6$ which is also minimum.

33. We observe the following properties of f .

Injectivity: Let $x, y \in R_0$ such that $f(x) = f(y)$. Then,

$$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$$

So, $f: R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that $f(x) = y$. Then,

$$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$$

Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all $y \in R_0$ (co-domain).

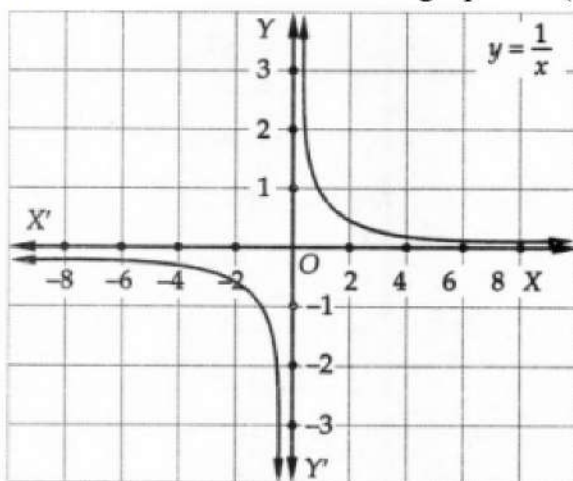
Thus, for each $y \in R_0$ (co-domain) there exists $x = \frac{1}{y} \in R_0$ (domain) such that

$$f(x) = \frac{1}{x} = y$$

So, $f: R_0 \rightarrow R_0$ is onto.

Hence, $f: R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of $f(x)$ as shown in fig.



Let us now consider $f: N \rightarrow R_0$ given by $f(x) = \frac{1}{x}$

For any $x, y \in N$, we find that

$$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$$

So, $f: N \rightarrow R_0$ is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N . So, $f: N \rightarrow R_0$ is not onto.

Thus, $f: N \rightarrow R_0$ is one-one but not onto.

OR

For $x_1, x_2 \in R$, consider

$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1}$$

$$\Rightarrow x_1 x_2^2 + x_1 = x_2 x_1^2 + x_2$$

$$\Rightarrow x_1 x_2 (x_2 - x_1) = x_2 - x_1$$

$$\Rightarrow x_1 = x_2 \text{ or } x_1 x_2 = 1$$

We note that there are point, x_1 and x_2 with $x_1 \neq x_2$ and $f(x_1) = f(x_2)$ for instance, if we

take $x_1 = 2$ and $x_2 = \frac{1}{2}$, then we have $f(x_1) = \frac{2}{5}$ and $f(x_2) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$. Hence f is not one-

one. Also, f is not onto for if so then for $1 \in R \exists x \in R$ such that $f(x) = 1$ which gives

$$\frac{x}{x^2 + 1} = 1. \text{ But there is no such } x \text{ in the domain } R, \text{ since the equation } x^2 - x + 1 = 0 \text{ does}$$

not give any real value of x .

34. Here the equation of two planes are: $\vec{r} \cdot (\hat{i} - \hat{j} + 2\hat{k}) = 5$ and $\vec{r} \cdot (3\hat{i} + \hat{j} + \hat{k}) = 6$.

Since the line is parallel to the two planes.

$$\therefore \text{Direction of line } \vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$$

$$= -3\hat{i} + 5\hat{j} + 4\hat{k}$$

$\therefore$ Equation of required line is

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k}) \dots\dots\dots (i)$$

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$

For this line to intersect the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k})$ we have

$$(1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$$

$$\Rightarrow \lambda = 1$$

$\therefore$ Point of intersection is $(4, -3, -1)$

OR

Here, it is given that

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

Here,

$\rightarrow$

$$a_1 = \hat{i} + 2\hat{j} + 3\hat{k}$$

$\rightarrow$

$$b_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$\rightarrow$

$$a_2 = 4\hat{i} + \hat{j}$$

$\rightarrow$

$$b_2 = 5\hat{i} + 2\hat{j} + \hat{k}$$

Thus,

$$\rightarrow \rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$$

$$= \hat{i}(3-8) - \hat{j}(2-20) + \hat{k}(4-15)$$

$$\therefore \vec{b_1} \times \vec{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$$

$$\begin{aligned} \therefore |\vec{b_1} \times \vec{b_2}| &= \sqrt{(-5)^2 + 18^2 + (-11)^2} \\ &= \sqrt{25 + 324 + 121} \\ &= \sqrt{470} \end{aligned}$$

$$\begin{aligned} \vec{a_2} - \vec{a_1} &= (4-1)\hat{i} + (1-2)\hat{j} + (0-3)\hat{k} \\ \therefore \vec{a_2} - \vec{a_1} &= 3\hat{i} - \hat{j} - 3\hat{k} \end{aligned}$$

Now, we have

$$\begin{aligned} (\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) &= (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k}) \\ &= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3)) \\ &= -15 - 18 + 33 \\ &= 0 \end{aligned}$$

Thus, the distance between the given lines is

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$$

$$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$$

$$\therefore d = 0 \text{ units}$$

As $d = 0$

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$$\Rightarrow \vec{L_1}: x\hat{i} + y\hat{j} + z\hat{k} = (i + 2j + 3\hat{k}) + \lambda(2i + 3\hat{j} + 4\hat{k})$$

$$\Rightarrow \vec{L_2}: x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$$

$$\Rightarrow \vec{L_1}: (x-1)\hat{i} + (y-2)\hat{j} + (z-3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$$

$$\Rightarrow \vec{L_2}: (x-4)\hat{i} + (y-1)\hat{j} + (z-0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$$

$$\Rightarrow \vec{L_1}: \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$$

$$\therefore \vec{L_2}: \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$$

General point on L1 is

$$x_1 = 2\lambda + 1, y_1 = 3\lambda + 2, z_1 = 4\lambda + 3$$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$.

$$\Rightarrow \frac{2\lambda + 1 - 4}{5} = \frac{3\lambda + 2 - 1}{2} = \frac{4\lambda + 3 - 0}{1}$$

$$\therefore \frac{2\lambda - 3}{5} = \frac{3\lambda + 1}{2}$$

$$\Rightarrow 4\lambda - 6 = 15\lambda + 5$$

$$\Rightarrow 11\lambda = -11$$

$$\Rightarrow \lambda = -1$$

Thus, $x_1 = 2(-1) + 1, y_1 = 3(-1) + 2, z_1 = 4(-1) + 3$

$$\Rightarrow x_1 = -1, y_1 = -1, z_1 = -1$$

Therefore, point of intersection of given lines is $(-1, -1, -1)$.

35. Given: $x^y + y^x = 1$

$$\text{Let } u = x^y + y^x = 1$$

$$\text{Let } u = x^y \text{ and } v = y^x$$

$$\text{Then, } u + v = 1$$

$$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$$

$$\text{For, } u = x^y$$

Taking log on both sides, we get

$$\log u = \log x^y$$

$$\Rightarrow \log u = y \cdot \log(x)$$

Now, differentiating both sides with respect to x

$$\frac{d}{dx}(\log u) = \frac{d}{dx}[y \cdot \log(x)]$$

$$\Rightarrow \frac{1}{u} \frac{du}{dx} = \left\{ y \cdot \frac{d}{dx}(\log x) + \log x \cdot \frac{d}{dx}(y) \right\}$$

$$\Rightarrow \frac{du}{dx} = u \left[y \cdot \frac{1}{x} + \log x \cdot \left(\frac{dy}{dx} \right) \right]$$

$$\Rightarrow \frac{du}{dx} = x^y \left[\frac{y}{x} + \log x \cdot \left(\frac{dy}{dx} \right) \right]$$

$$\text{For } v = y^x$$

Taking log on both sides, we get

$$\log v = \log y^x$$

$$\Rightarrow \log v = x \cdot \log(y)$$

Now, differentiate both sides with respect to x

$$\frac{d}{dx}(\log v) = \frac{d}{dx}[x \cdot \log(y)]$$

$$\Rightarrow \frac{1}{v} \frac{dv}{dx} = \left\{ x \cdot \frac{d}{dx}(\log y) + \log y \cdot \frac{d}{dx}x \right\}$$

$$\Rightarrow \frac{dv}{dx} = v \left[x \cdot \frac{1}{y} \cdot \frac{dy}{dx} + \log y \cdot \left(\frac{dx}{dx} \right) \right]$$

$$\Rightarrow \frac{dv}{dx} = y^x \left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y \right]$$

because, $\frac{du}{dx} + \frac{dv}{dx} = 0$

$$\text{So, } x^y \left[\frac{y}{x} + \log x \cdot \left(\frac{dy}{dx} \right) \right] + y^x \left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y \right] = 0$$

$$\Rightarrow \left(x^y \log x + x y^{x-1} \right) \cdot \frac{dy}{dx} + \left(y x^{y-1} + y^x \log y \right) = 0$$

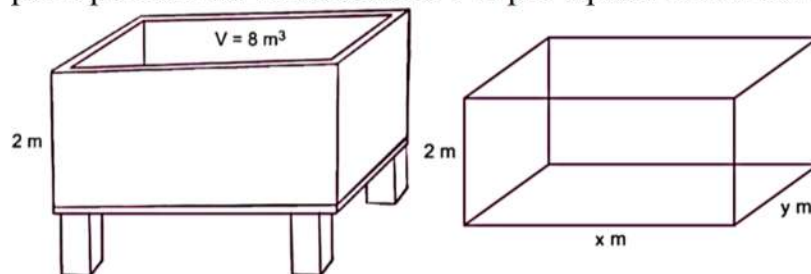
$$\Rightarrow \left(x^y \log x + x y^{x-1} \right) \cdot \frac{dy}{dx} = - \left(y x^{y-1} + y^x \log y \right)$$

$$\frac{dy}{dx} = - \frac{\left(y x^{y-1} + y^x \log y \right)}{\left(x^y \log x + x y^{x-1} \right)}$$

Section E

36. Read the text carefully and answer the questions:

On the request of villagers, a construction agency designs a tank with the help of an architect. Tank consists of a rectangular base with rectangular sides, open at the top so that its depth is 2 m and volume is 8 m^3 as shown below. The construction of the tank costs ₹70 per sq. metre for the base and ₹45 per square metre for sides.



(i) Since 'C' is cost of making tank

$$\therefore C = 70xy + 45 \times 2(2x + 2y)$$

$$\Rightarrow C = 70xy + 90(2x + 2y)$$

$$\Rightarrow C = 70xy + 180(x + y) \quad [\because 2 \cdot x \cdot y = 8 \Rightarrow y = \frac{8}{2x} \Rightarrow y = \frac{4}{x}]$$

$$\Rightarrow C = 70x \times \frac{4}{x} + 180 \left(x + \frac{4}{x} \right)$$

$$\Rightarrow C = 280 + 180 \left(x + \frac{4}{x} \right)$$

(ii) $x \cdot y = 4$

Volume of tank = length $\times$ breadth $\times$ height (Depth)

$$8 = x \cdot y \cdot 2$$

$$\Rightarrow 2xy = 8 \Rightarrow xy = 4$$

(iii) For maximum or minimum

$$\frac{dC}{dx} = 0$$

$$\frac{d}{dx}(280 + 180(x + \frac{4}{x})) = 0 \Rightarrow 180 \left(1 + 4 \left(-\frac{1}{x^2} \right) \right) = 0$$

$$\Rightarrow 180 \left(1 - \frac{4}{x^2} \right) = 0 \Rightarrow 1 - \frac{4}{x^2} = 0$$

$$\Rightarrow \frac{4}{x^2} = 1 \Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$$\Rightarrow x = 2 \text{ (length can never be negative)}$$

OR

$$\text{Now, } \frac{d^2C}{dx^2} = 180 \left(+\frac{8}{x^3} \right)$$

$$\Rightarrow \left. \frac{d^2C}{dx^2} \right|_{x=2} = 180 \times \frac{8}{8} = 180 = +ve$$

Hence, to minimize C, $x = 2m$

37. Read the text carefully and answer the questions:

Three car dealers, say A, B and C, deals in three types of cars, namely Hatchback cars, Sedan cars, SUV cars. The sales figure of 2019 and 2020 showed that dealer A sold 120 Hatchback, 50 Sedan, 10 SUV cars in 2019 and 300 Hatchback, 150 Sedan, 20 SUV cars in 2020; dealer B sold 100 Hatchback, 30 Sedan, 5 SUV cars in 2019 and 200 Hatchback, 50 Sedan, 6 SUV cars in 2020; dealer C sold 90 Hatchback, 40 Sedan, 2 SUV cars in 2019 and 100 Hatchback, 60 Sedan, 5 SUV cars in 2020.



(i) *Hatchback Sedan SUV*

$$\begin{array}{l} A \\ B \\ C \end{array} \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$$

In 2019, dealer A sold 120 Hatchbacks, 50 Sedans and 10 SUV; dealer B sold 100 Hatchbacks, 30 Sedans and 5 SUVs and dealer C sold 90 Hatchbacks, 40 Sedans and 2 SUVs.

∴ Required matrix, say P, is given by

Hatchback Sedan SUV

$$P = \begin{array}{l} A \\ B \\ C \end{array} \begin{bmatrix} 120 & 50 & 10 \\ 100 & 30 & 5 \\ 90 & 40 & 2 \end{bmatrix}$$

In 2020, dealer A sold 300 Hatchbacks, 150 Sedans, 20 SUVs dealer B sold 200 Hatchbacks, 50 sedans, 6 SUVs dealer C sold 100 Hatchbacks, 60 sedans, 5 SUVs.

∴ Required matrix, say Q, is given by

Hatchback Sedan SUV

$$Q = \begin{array}{l} A \\ B \\ C \end{array} \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$$

(ii) *Hatchback Sedan SUV*

$$\begin{array}{l} A \\ B \\ C \end{array} \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$$

In 2020, dealer A sold 300 Hatchback, 150 Sedan, 20 SUV dealer B sold 200 Hatchback, 50 sedan, 6 SUV dealer C sold 100 Hatchback, 60 sedan, 5 SUV.

∴ Required matrix, say Q, is given by

Hatchback Sedan SUV

$$Q = \begin{array}{l} A \\ B \\ C \end{array} \begin{bmatrix} 300 & 150 & 20 \\ 200 & 50 & 6 \\ 100 & 60 & 5 \end{bmatrix}$$

(iii) Total number of cars sold in two given years, by each dealer, is given by

Hatchback Sedan SUV

$$P + Q = \begin{array}{l} A \\ B \\ C \end{array} \begin{bmatrix} 120 + 300 & 50 + 150 & 10 + 20 \\ 100 + 200 & 30 + 50 & 5 + 6 \\ 90 + 100 & 40 + 60 & 2 + 5 \end{bmatrix}$$

$$\begin{array}{c}
 \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\
 A \quad \begin{bmatrix} 420 & 200 & 30 \end{bmatrix} \\
 = B \quad \begin{bmatrix} 300 & 80 & 11 \end{bmatrix} \\
 C \quad \begin{bmatrix} 190 & 100 & 7 \end{bmatrix}
 \end{array}$$

OR

The amount of profit in 2020 received by each dealer is given by the matrix

$$\begin{array}{c}
 \text{Hatchback} \quad \text{Sedan} \quad \text{SUV} \\
 A \quad \begin{bmatrix} 300 & 150 & 20 \end{bmatrix} \quad \begin{bmatrix} 50000 \end{bmatrix} \\
 B \quad \begin{bmatrix} 200 & 50 & 6 \end{bmatrix} \quad \begin{bmatrix} 100000 \end{bmatrix} \\
 C \quad \begin{bmatrix} 100 & 60 & 5 \end{bmatrix} \quad \begin{bmatrix} 200000 \end{bmatrix}
 \end{array}$$

$$\begin{array}{c}
 A \quad \begin{bmatrix} 15000000 + 15000000 + 4000000 \end{bmatrix} \\
 = B \quad \begin{bmatrix} 10000000 + 5000000 + 1200000 \end{bmatrix} \\
 C \quad \begin{bmatrix} 5000000 + 6000000 + 1000000 \end{bmatrix}
 \end{array}$$

$$\begin{array}{c}
 A \quad \begin{bmatrix} 34000000 \end{bmatrix} \\
 = B \quad \begin{bmatrix} 16200000 \end{bmatrix} \\
 C \quad \begin{bmatrix} 12000000 \end{bmatrix}
 \end{array}$$

38. Read the text carefully and answer the questions:

To teach the application of probability a maths teacher arranged a surprise game for 5 of his students namely Govind, Girish, Vinod, Abhishek and Ankit. He took a bowl containing tickets numbered 1 to 50 and told the students go one by one and draw two tickets simultaneously from the bowl and replace it after noting the numbers.



- (i) Required probability = P(one ticket with prime number and other ticket with a multiple of 4)

$$= 2 \left(\frac{15}{50} \times \frac{12}{49} \right) = \frac{36}{245}$$

- (ii) P(First ticket shows an even number and second ticket shows an odd number)

$$= \frac{25}{50} \times \frac{25}{49} = \frac{25}{98}$$